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LETTER TO THE EDITOR

The single impurity problem with interactions in two dimensions

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Abstract. We study the behaviour of a single impurity potential V_{imp} within a combined method of renormalization group and bosonization in two dimensions. For repulsive interactions the $2K_F$ backscattering components of the impurity potential are enhanced with respect to the other angles. We estimate the corrections to the conductivity as a function of E_F/T. For $E_F/T \to \infty$ we obtain similar results as in the non-interacting case. At finite temperatures, we obtain results controlled by the interactions. Here we identify a range of temperatures in which the conductivity increases with the lowering of the temperature.

The problem of interaction and disorder in low-dimensional systems has become a popular subject. Theoretically it has been shown in one dimension that for a single impurity, interactions can lead to delocalization [1]. For repulsive interactions an enhancement [2] of the density of states might arise in addition to localization. For many impurities, a metal insulator transition driven by attractive interactions has been obtained in one dimension [3]. In two dimensions, the common belief was that non metal–insulator transition is possible. This belief has been challenged by Finkelstein [4] who emphasized that two-body interactions become important for $T \to 0$ and might affect the transition.

Recent experiments [5] confirm the possibility of a metal–insulator transition in two dimensions when the two-body interaction is large and the Fermi energy E_F is comparable to the temperature T.

We will study within the renormalization group the problem of a single impurity in two dimensions in the presence of a short-range two-body interaction. In this first attempt we will ignore the Cooperon interactions and spin effects. We will derive a set of differential equations for the two-body interaction and the impurity strength as a function of the scale, $\ell \equiv \ell(t) = \mathrm{Log}(1/t)$, $t = T/E_F$. We find the following results. In the absence of interaction, the stiffness parameter K is $K = 1$. In the presence of the two-body interaction $K \neq 1$, $K = K(\ell)$ we obtain $K(\ell) = 1 - \hat{U}_0\, \mathrm{e}^{-\ell}$ where $\hat{U}_0$ is the dimensionless short-range two-body interaction (measured in units of E_F) at the microscopic scale. The meaning of these results is that far from the Fermi surface the two-body interaction gives rise to a non-Fermi liquid behaviour. In the limit $T \to 0$ ($\ell \to \infty$) we recover the Fermi liquid theory with $K = 1$. At $T \neq 0$ we obtain $K = K(t) = 1 - \hat{U}_0(T/E_F)$. The temperature dependence of K will affect the scaling of the impurity potential V_{imp} which in the presence

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of the two-dimensional Fermi surface (FS) is described by an angular scattering potential $V(\varphi)$, $0 \leqslant \varphi \leqslant 2\pi$. We treat separately the backscattering case $\varphi = \pi$, $V(\pi) = g$. The scaling of g will be different from $V(\varphi \neq \pi)$. In the absence of interaction, the effect of g will be of the order of $1/N$ with respect to the total scattering and will be negligible. The presence of interaction will suppress the angular potential $V(\varphi)$ and will enhance (for repulsive interactions) the backscattering term g. The physical properties will depend on the number of channels N. We introduce the cutoff $\Lambda = K_F$ and the scaling factor $b = \mathrm{e}^{\ell}$. We have

$$N(\ell) = \frac{2\pi K_F}{K_F/b} = 2\pi b = 2\pi\,\mathrm{e}^{\ell}.$$

We stop scaling at $\ell = \ell(t)$ which gives us $N = N(t) = 2\pi(E_F/T)$. For a standard Fermi system we have $E_F/T \to \infty$ and $N \to \infty$. The relations are not applicable to the recent experiments [5] where $E_F \sim T$.

The single impurity problem in two dimensions has been studied in the absence of interactions [12]. The impurity potential obeys the scaling equation $\mathrm{d}V/\mathrm{d}\ell = -mV^2/2\pi^2$. In the presence of a Fermi surface the result obtained in [12] is not valid. This can be seen from the resonance condition

$$\frac{1}{V} = P\int \frac{\rho(E')}{E - E'}\,\mathrm{d}E'$$

(where P is the principal value). In two dimensions for a symmetric cutoff $\pm\Lambda$ around E_F we have:

$$\frac{1}{V} = \frac{m}{2\pi^2} P\int_{-\Lambda}^{\Lambda} \frac{\mathrm{d}z}{\omega - z} \qquad \omega = E - E_F.$$

Due to the symmetric cutoff we obtain $\mathrm{d}V/\mathrm{d}\ell = 0$. This result means that there are no second-order corrections. As in one dimension, the interaction will modify the scaling dimensions of the impurity potential. Therefore, we expect that the interaction will give rise to the scaling $\mathrm{d}V/\mathrm{d}\ell = F(K)V$, where the function $F(K)$ depends on the interaction and scattering angle. In particular, for the $2K_F$ backscattering we obtain the same result as in one dimension, $F(K) = 1 - K$. Contrary to the one-dimensional case, K is not fixed and depends on the scale. For the non $2K_F$ scattering we obtain $F(K) = 1 - \frac{1}{2}(K + 1/K)$. The combination K and $1/K$ appears since the scattering involves, in the bosonic language, the field θ and the dual field ϕ. Explicit calculation will show that the angular scattering (different to $2K_F$) has the form:

$$V(\ell(t)) \approx \tilde{V}(0)\exp\left\{\frac{\hat{U}_0^2}{4}\left(\frac{T}{E_F}\right)^2\right\}.$$

For the $2K_F$ backscattering we find

$$g(\ell(t)) = \hat{g}(0)\exp\left\{-\hat{U}_0\left(\frac{T}{E_F}\right)\right\}.$$

We observe that for $T \to 0$ the angular scattering $V(\ell(t))$ is reduced and the backscattering terms $g(\ell(t))$ are enhanced. Therefore if the angular scattering $V(\ell(t))$ is dominant we expect to have a metallic behaviour and the insulating behaviour is determined by $g(\ell(t))$. We conclude that the competition between the two terms will determine the physical behaviour.

We remark that the $2K_F$ backscattering term is similar to the one-dimensional case with the major difference that the weight of the $2K_F$ terms is of the order of $1/N$ with respect to the rest of the scatterings.

Here we recognize the similarity with the system of coupled Luttinger chains ($t_\perp$ is the interchain hopping and $t_\parallel$ the intrachain hopping). For $t_\parallel > t_\perp$ and finite temperature we find that the one-dimensional Luttinger theory is valid at temperatures $T > t_\perp$. The presence of a single impurity with matrix elements $V_{n,m}$ ($n \neq m$ being the chain index) and g the $2K_F$ backscattering term gives rise to the scaling equations

$$\frac{\mathrm{d}V_{n,m}}{\mathrm{d}\ell} = \left[1 - \frac{1}{2}\left(K + \frac{1}{K}\right)\right] V_{n,m} \qquad \frac{\mathrm{d}g}{\mathrm{d}\ell} = [1-K]g$$

where $K \neq 1$ is the Luttinger parameter. These equations are valid for $T \geqslant t_\perp$, $0 < \ell \leqslant \mathrm{Log}(t_\parallel/t_\perp)$. When $T < t_\perp$ tunnelling will destroy the Luttinger liquid behaviour making the scaling equations invalid.

In the remainder we will introduce a model for a single impurity in two dimensions in the presence of interactions.

We replace the fermion field $\psi(x) = \sum_{n=1}^{N/2} \psi_n(x)$ by:

$$\psi(\boldsymbol{x}) \equiv \sum_{n=1}^{N/2} (\mathrm{e}^{\mathrm{i}K_F \hat{n}\cdot\boldsymbol{x}} R_n(\boldsymbol{x}) + \mathrm{e}^{-\mathrm{i}K_F \hat{n}\cdot\boldsymbol{x}} L_n(\boldsymbol{x})). \tag{1a}$$

The Fermi surface (FS) is parametrized in terms of N fermions or $N/2$ pairs of right and left movers:

$$R_n(\boldsymbol{x}) = \hat{R}_n(x_\parallel) Z_n(x_\perp) \qquad L_n(\boldsymbol{x}) = \hat{L}_n(x_\parallel) Z_n(x_\perp). \tag{1b}$$

$\hat{R}_n(x_\parallel)$ and $\hat{L}_n(x_\parallel)$, $x_\parallel = \hat{n}\cdot\boldsymbol{x}$ represents a $1+1$ Dirac fermion [6, 8] which is bosonized as in the one-dimensional case [7]. $Z_n(x_\perp)$ is a scalar function which ensures the conservation of momentum in the transversal direction. The patch is characterized by two cutoffs $K_F/b_0 \times K_F/b_0$. The numbers of patches $N(b_0) = 2\pi b_0$, $b_0 = \mathrm{e}^{\ell_0}$ is a function of the cutoff K_F/b_0 (b_0 is the reduction of the cutoff $\Lambda = K_F$). Keeping only the linear dispersion $\epsilon(\boldsymbol{K}) - E_F \simeq v_F \hat{n}\cdot\boldsymbol{q}$, we obtain the Euclidean representation of the free fermion action [9, 10] at the scale b_0

$$S_0 = \sum_{n=1}^{N/2} \int \mathrm{d}\tau \int \mathrm{d}^d x \{R_n^+(\boldsymbol{x},\tau)[\partial_\tau - v_F \hat{n}\cdot\boldsymbol{\partial}]R_n(\boldsymbol{x},\tau) + L_n^+(\boldsymbol{x},\tau)[\partial_\tau + v_F \hat{n}\cdot\boldsymbol{\partial}]L_n(\boldsymbol{x},\tau)\}. \tag{2a}$$

The action in (2a) is invariant [9, 10] under $K_F \to K_F/b_0$ and $x = x' b_0$, $\tau = \tau' b_0$ if R_n and L_n obey the scaling equations $R_n(\boldsymbol{x},\tau) = b_0^{-d/2} R_n(\boldsymbol{x}',\tau')$, $L_n(\boldsymbol{x},\tau) = b_0^{-d/2} L_n(\boldsymbol{x}',\tau')$. Next we consider a short range potential of strength U. Using the decomposition given in (1a) we find [9, 10]

$$S_{int} = \frac{K_F^{1-d}}{2b_0^{d-1}} \sum_{n=1}^{N}\sum_{m=1}^{N} \int \mathrm{d}\tau \int \mathrm{d}^d x \{\hat{U}(n,m)\psi_n^+(\boldsymbol{x})\psi_m^+(\boldsymbol{x})\psi_m(\boldsymbol{x})\psi_n(\boldsymbol{x})\}. \tag{2b}$$

The two-body potential is expressed in terms of the dimensionless interaction, $U(n,m) = \Lambda^{1-d}\hat{U}(n,m)$ with $\Lambda = K_F$, $\hat{U}(n,m)$ being the dimensionless interaction. The factor b in (2b) appears as a result of the reduced cutoff K_F/b_0 and rescaling of the fields (the cutoff K_F/b_0 is restored to K_F by rescaling the fields ψ_n and replacing $x = x' b_0$, $\tau = \tau' b_0$). The factor b_0^{d-1} is proportional to $N = N(b_0)$, the number of patches. This allows us to treat the problem as a large N problem. This holds for all the channels expect the Cooperon one [10]. For the Cooperon channel we will have an extra summation which will cancel the $1/N$ factor. Here we will ignore these effects. We do this for reasons of simplicity. (We expect that the Cooperon will play an important role since *independent* of the original two-body interactions in the limit $T \to 0$ we will have *attractive* Cooperon channels.)

Next we will consider the impurity potential $\hat{V}_{imp}(\boldsymbol{x}) = V_{imp}\delta^{(d)}(\boldsymbol{x})$. Using (1$a$) we construct the action S_{imp} for the impurity potential

$$S_{imp} = -\frac{K_F^{1-d}}{b_0^{d-1}} \int \mathrm{d}\tau \int \mathrm{d}^d x \delta^{(d)}(\boldsymbol{x}) \sum_{n=1}^{N/2} \sum_{m=1}^{N/2} \{\hat{V}^{(F)}(n,m)[R_n^+(\boldsymbol{x})R_m(\boldsymbol{x}) + L_n^+(\boldsymbol{x})L_m(\boldsymbol{x})] + \hat{V}^{(B)}(n,m)[R_n^+(\boldsymbol{x})L_m(\boldsymbol{x}) + L_n^+(\boldsymbol{x})R_m(\boldsymbol{x})]\}. \tag{2c}$$

Dimensional analysis gives: $V^{(F)}(n,m) = K_F^{1-d}\hat{V}^{(F)}(n,m)$, $V^{(B)}(n,m) = K_F^{1-d}\hat{V}^{(B)}(n,m)$. $V^{(F)}(n,m)$ and $V^{(B)}(n,m)$ are the angular components of the potential $V(\varphi)$. $V(\varphi) \equiv V^{(F)}(n,m)$, $0 < \varphi \leqslant \pi$, $n \cdot m \equiv \cos\varphi$ and $V(\varphi) \equiv V^{(B)}(n,m)$, $\pi < \varphi \leqslant 2\pi$. We will bosonize the action:

$$S = S_0 + S_{int} + S_{imp}.$$

Following the method described in [8, 11] we define:

$$: R_n^+(\boldsymbol{x}+\boldsymbol{\epsilon})R_n(\boldsymbol{x}+\boldsymbol{\epsilon}) : \underset{\epsilon\to 0}{=} \left(\frac{K_F}{2\pi}\right)^{\frac{d-1}{2}} \frac{1}{\sqrt{\pi}} \hat{n} \cdot \boldsymbol{\partial}\theta_n^{(R)}(\boldsymbol{x})$$
$$: L_n^+(\boldsymbol{x}+\boldsymbol{\epsilon})L_n(\boldsymbol{x}+\boldsymbol{\epsilon}) : \underset{\epsilon\to 0}{=} \left(\frac{K_F}{2\pi}\right)^{\frac{d-1}{2}} \frac{1}{\sqrt{\pi}} \hat{n} \cdot \boldsymbol{\partial}\theta_n^{(L)}(\boldsymbol{x}). \tag{3a}$$

We obtain the Kac–Mody algebra with the anomaly $(K_F/2\pi)^{d-1}$. Following the one-dimensional scheme we replace the chiral fields $\theta_n^{(R)}$ and $\theta_n^{(L)}$ by the boson field θ_n and the dual field ϕ_n

$$\theta_n(\boldsymbol{x}) = \theta_n^{(R)}(\boldsymbol{x}) + \theta_n^{(L)}(\boldsymbol{x}) \qquad \phi_n(\boldsymbol{x}) = -\theta_n^{(R)}(\boldsymbol{x}) + \theta_n^{(L)}(\boldsymbol{x}). \tag{3b}$$

The d dimensional bosonic fields $\theta_n(\boldsymbol{x})$ are related to the one-dimensional field $\theta_n(x_\parallel)$ by the scalar function $Z_n(x_\perp)$

$$\theta_n(\boldsymbol{x}) = \hat{\theta}_n(x_\parallel)Z_n(x_\perp) \tag{3c}$$

where $\langle Z_n(x_\perp)Z_m(x'_\perp)\rangle = \delta_{n,m}\delta_{K_F}^{(d-1)}(x_\perp - x'_\perp)$. Making use of the relations (3a)–(3c) we bosonize the action S. Neglecting the Cooperon part, we define $\tilde{S}_0 = S_0 + S_{int}^{(F)}$. The bosonic form of $\hat{S}$ is given by:

$$\tilde{S}_0 = \sum_{n=1}^{N/2} \int \frac{\mathrm{d}\omega}{2\pi} \int \frac{\mathrm{d}^d q}{(2\pi)^d} \left\{\frac{1}{2}\theta_n(\boldsymbol{q},\omega)\left[\frac{\omega^2 + \tilde{v}_F^2(\hat{n}\cdot\boldsymbol{q})^2}{v_F K(|\omega/q|,\tilde{U})}\right]\theta_n(-\boldsymbol{q},-\omega)\right\} \tag{4a}$$

where v_F is the renormalized Fermi velocity and K describes the two-body renormalization of the stiffness parameter:

$$K(\ell) \approx 1 - \hat{U}(\ell) = 1 - \frac{\hat{U}_0}{(b(\ell))^{d-1}} \qquad b(\ell) = \mathrm{e}^{\ell}. \tag{4b}$$

Next we bosonize the impurity action given in (2d). We will separate the diagonal term from $\hat{V}^F(n,m)$ and $\hat{V}^{(B)}(n,m)$, $n \neq m$ and the diagonal term $n = m$ will be the $\hat{g}(n)$ backscattering term. The rule for bosonization will be as in one dimension:

$$R_n(\boldsymbol{x}) = \sqrt{\frac{2K_F}{\pi^2}} Z_n(x_\perp)\,\mathrm{e}^{\mathrm{i}\sqrt{4\pi}\hat{\theta}^{(R)}_n}(x_\parallel) \qquad L_n(\boldsymbol{x}) = \sqrt{\frac{2K_F}{\pi^2}} Z_n(x_\perp)\,\mathrm{e}^{-\mathrm{i}\sqrt{4\pi}}\hat{\theta}_n^{(R)}(x_\parallel). \tag{4c}$$

Using (4c) we obtain the bosonic form of S_{imp}:

$$S_{imp} = -\frac{K_F^{1-d}}{b^{d-1}}\left(\frac{2K_F}{\pi}\right)\int \mathrm{d}\tau \int \mathrm{d}^d x\, \delta_{K_F}^{(d)}(\boldsymbol{x}) \sum_{n=1}^{N/2}\sum_{m=1}^{N/2} Z_n(x_\perp) Z_n(x_\perp) Z_m(x_\perp) Z_m(x_\perp)$$
$$\times\{(1-\delta_{n,m})[\hat{V}^{(F)}(\hat{n},\hat{m})\cos\sqrt{\pi}(\hat{\theta}_n(x_\parallel,\tau) - \hat{\theta}_m(x_\parallel,\tau))\cos\sqrt{\pi}(\hat{\phi}(x_\parallel\tau)$$
$$-\hat{\phi}_m(x_\parallel\tau)) + \hat{V}^{(B)}(n,m)\cos(\hat{\theta}_m(x_\parallel,\tau) + \hat{\theta}_m(x_\parallel,\tau))\cos\sqrt{\pi}(\hat{\phi}_n(x_\parallel,\tau)$$
$$-\hat{\phi}_m(x_\parallel,\tau))] + \delta_{n,m}[\hat{g}(n)\cos\sqrt{4\pi}\hat{\theta}_n(x_\parallel,\tau)]\}. \qquad (4d)$$

In (4d) we have separated the $2K_F$ backscattering term from the rest and have neglected the forward term of the form $\sum_n (1/\sqrt{\pi})\hat{n}\cdot\boldsymbol{\partial}\theta_n(\boldsymbol{x},\tau)$. Such a term can be neglected since it corresponds to the 'first-order derivative' which is eliminated by integration. Next we perform renormalization group (RG) calculation of the action: $S = \tilde{S}_0 + S_{imp}$. We perform a differential RG calculation following the method given in [9].

We renormalize the action $S = \tilde{S}_0 + S_{imp}$:

$$S = S_< + \delta^{(1)}S + \tfrac{1}{2}\delta^{(2)}S = \tilde{S}_{0,<} + S_{imp,<} + \tfrac{1}{2}\delta^{(2)}\tilde{S}_0 + \delta^{(1)}S_{imp} + \tfrac{1}{2}\delta^{(2)}S_{imp}.$$

$S_<$ represents the action at the cutoff $K_F\,\mathrm{e}^{-\ell}$ and $\delta^{(1)}S + \frac{1}{2}\delta^{(2)}S$ represents the second-order differential correction to the action. Integrating the fields in the differential shell $d\Lambda = K_F - K_F\,\mathrm{e}^{-\ell}$ we obtain the effective action

$$S_{eff} = S_< + \tfrac{1}{2}\langle\delta^{(2)}\delta_{imp}\rangle_{\delta\tilde{S}_0} - \tfrac{1}{2}\langle(\delta^{(1)}S_{imp})^2\rangle_{\delta\tilde{S}_0}. \qquad (5a)$$

In (5a) $\langle\,\rangle_{\delta\tilde{S}_0}$ represents the expectation value with respect to the Gaussian action given by (4a) in the shell $d\Lambda$. Next, we rescale S_{eff} and obtain the renormalized action $\hat{S}$ with b_0 replaced by $b = b(\ell) = b_0(1 + d\Lambda/K_F)$. We obtain the following set of RG equations

$$\frac{\mathrm{d}v_F}{\mathrm{d}\ell} \simeq 0 \qquad (6a)$$

$$k(\ell) = 1 - \hat{U}(\ell) \qquad \hat{U}(\ell) = \hat{U}_0\,\mathrm{e}^{-\ell(d-1)} \qquad (6b)$$

$$\frac{\mathrm{d}\hat{V}^{(F)}_{(n,m)}}{\mathrm{d}\ell} = \left[1 - \frac{1}{2}\left(K + \frac{1}{K}\right)\right]\hat{V}^{(F)}(n,m) \qquad (6c)$$

$$\frac{\mathrm{d}\hat{V}^{(B)}_{(n,m)}}{\mathrm{d}\ell} = \left[1 - \frac{1}{2}\left(K + \frac{1}{K}\right)\right]\hat{V}^{(B)}_{(n,m)} \qquad (6d)$$

$$\frac{\mathrm{d}\hat{g}(n)}{\mathrm{d}\ell} = [1 - K]\hat{g}(n). \qquad (6e)$$

We assume that at the microscopic scale $\hat{g}(n) = \hat{V}^{(F)}(n,m) = \hat{V}^{(B)}(n,m) \equiv \hat{V}$ for all the channels.

We replace in equation (6b), $\frac{1}{2}(K + 1/K) \simeq 1 + \hat{U}^2/2$, $1 - K \simeq \hat{U}$ by $\hat{U} = \hat{U}(\ell)$. We obtain instead of (6a)–(6e) the new set:

$$\frac{\mathrm{d}\hat{U}}{\mathrm{d}\ell} = -(d-1)\hat{U} \qquad d = 2 \qquad \hat{U}(\ell = 0) = \hat{U}_0 \qquad (7a)$$

$$\frac{\mathrm{d}\hat{V}}{\mathrm{d}\ell} = -\frac{1}{2}\hat{U}^2\hat{V} \qquad (7b)$$

$$\frac{\mathrm{d}\hat{g}}{\mathrm{d}\ell} = \hat{U}\hat{g}. \qquad (7c)$$

Next we investigate (7a)–(7c) for different cases.

(a) The fixed point behaviour is obtained in the limit $T \to 0$ ($\ell \to \infty$). The set of equations (7*a*)–(7*c*) has the fixed point $\hat{U}(\ell \to \infty) = 0$ giving rise to the marginal behaviour $\mathrm{d}V/\mathrm{d}\ell = 0$, $\mathrm{d}g/\mathrm{d}\ell = 0$ obtained for the non-interacting case.

(b) The general solution. Integrating the set (7*a*)–(7*c*) with the initial conditions, $\hat{U}_0$, $\hat{V}(0)$ and $\hat{g}(0)$ gives for $d = 2$:

$$\hat{U}(\ell) = \hat{U}_0\,\mathrm{e}^{-\ell} \tag{8a}$$

$$\hat{V}(\ell) = \hat{V}(0)\exp\left\{-\frac{\hat{U}_0^2}{4}(1-\mathrm{e}^{-2\ell})\right\} \tag{8b}$$

$$\hat{g}(\ell) = \hat{g}(0)\exp\{\hat{U}_0(1-\mathrm{e}^{-\ell})\}. \tag{8c}$$

We introduce in (8*a*)–(8*c*) the scaling function $\mathrm{e}^{-\ell} = T/E_F$ and obtain the temperature dependence of the scattering potential. In the high-temperature limit, we replace $1-\mathrm{e}^{-\ell} \simeq \ell = \mathrm{Log}(E_F/T)$ and $1-\mathrm{e}^{2\ell} \sim 2\,\mathrm{Log}(E_F/T)$.

As a result we obtain:

$$\hat{V}(\ell) \simeq \hat{V}(0)\left(\frac{T}{E_F}\right)^{\hat{U}_0^2/2} \qquad \hat{g}(\ell) \simeq \hat{g}(0)\left(\frac{T}{E_F}\right)^{\hat{U}_0}. \tag{9a}$$

Equation (9*a*) shows that for repulsive interaction, $\hat{U}_0 > 0$, lowering the temperature enhances the $2K_F$ backscattering term $g(\ell)$ and reduces $\hat{V}(\ell)$ the angular scattering. In the limit $T \to 0$, $\mathrm{e}^{-\ell} \sim T/E_F \to 0$ and we obtain:

$$\hat{V}(\ell(t)) = \tilde{V}(0)\exp\left\{\frac{\hat{U}_0^2}{4}\left(\frac{T}{E_F}\right)^2\right\} \sim \tilde{V}(0)\left[1+\frac{\hat{U}_0^2}{4}\left(\frac{T}{E_F}\right)^2\cdots\right]$$

and

$$\hat{g}(\ell(t)) \approx \tilde{g}(0)\exp\left\{-\hat{U}_0\left(\frac{T}{E_F}\right)\right\} \sim \tilde{g}(0)\left[1-\hat{U}_0\left(\frac{T}{E}\right)\right] \tag{9b}$$

where

$$\tilde{V}(0) = \hat{V}(0)\exp\left\{-\frac{\hat{U}_0^2}{4}\right\} \qquad \text{and} \qquad \tilde{g}(0) = \hat{g}(0)\exp\{\hat{U}_0\}.$$

Equations (9*a*) and (9*b*) show that, independent of the sign of the two-body interaction, the angular potential $\hat{V}(\varphi)\varphi \neq \pi$ decreases with the lowering of the temperature. On the other hand, for $\varphi = \pi$, $\hat{g}$ grows with decreasing temperature ($\hat{U}_0 > 0$) and becomes irrelevant for $\hat{U}_0 < 0$ (attractive interaction). In order to see the effect of the scattering, we compute the scattering rate $1/\tau_{imp}$ for the single impurity using the Fermi–Golden rule. Since we consider only one impurity, we have no multiple scattering and have no way to compute the conductivity. To overcome this difficulty, we assume that our system has an initial conductivity G_0, and we want to find what is the change in the conductivity due to a single impurity.

We assume that G_0 is the conductivity of a system of size L. The effect of the impurity is to replace L by an effective length L_{eff}

$$\frac{1}{L_{eff}} = \frac{1}{L} + \frac{1}{\ell_{imp}}$$

(ℓ_{imp} is the elastic mean free path, $\ell_{imp} = v_F\tau_{imp}$ for the lifetime τ_{imp}). Then

$$\frac{\delta G}{G_0} = \frac{G-G_0}{G_0} = \frac{-L/\ell_{imp}}{1+L/\ell_{imp}}. \tag{10a}$$

We obtain $1/\tau_{imp}$ using the Golden rule

$$\frac{1}{\tau_{imp}(n)} = \frac{2\pi}{h}\left\{\sum_{m=1}^{N/2}\left|V^{(F)}_{(n,m)}\left(\frac{K_F}{2\pi}\right)^{d-1}\right|^2 + \sum_{m=1}^{N/2}\left|V^{(B)}_{(n,m)}\left(\frac{K_F}{2\pi}\right)^{d-1}\right|^2 + \left|g(n)\left(\frac{K_F}{2\pi}\right)^{d-1}\right|^2\right\}\mathcal{D}\left(E_F;\frac{K_F}{b}\right) \qquad (10b)$$

where $\tau_{imp}(n)$ is the lifetime of the channel n (in the direction of the external field) and $\mathcal{D}(E_F; K_F/b)$ is the two-dimensional density of states per unit area and energy at the scale K_F/b such that $N = 2\pi b$, $b = \mathrm{e}^{\ell}$. We introduce the initial values for the impurities $\hat{V}(0) = 2\hat{V}_{imp}$, $\hat{V}_{imp} = \hat{V}^{(F)} = \hat{V}^{(B)}$ and $\hat{g}(0) = \hat{V}_{imp}$. We find from (10$b$) (replacing $\hat{V}_{imp} \to e\hat{V}_{imp}$ where e is the charge):

$$\frac{1}{\tau_{imp}} = \frac{e^2}{h}\left(\frac{K_F}{v_F}\right)|\hat{V}_{imp}|^2\left[\left(\frac{\hat{V}(\ell)}{\hat{V}(0)}\right)^2 + \mathrm{e}^{-\ell}\left(\frac{\hat{g}(\ell)}{\hat{g}(0)}\right)^2\right]. \qquad (10c)$$

In (10c) we observe that the contribution of the backscattering is reduced by a factor of $1/N \simeq \mathrm{e}^{-\ell} = T/E_F$. We substitute the solutions of $\hat{V}(\ell)$ and $\hat{g}(\ell)$ given in (9a) and (9b).

Using (10a) we compute the corrections to the conductivity $\delta G/G_0$ as a function of $-L/\ell_{imp}$ for high temperatures using (9a):

$$-\frac{L}{\ell_{imp}} = -\frac{e^2}{h}(K_F L)\left|\frac{\hat{V}_{imp}}{v_F}\right|^2\left[\left(\frac{T}{E_F}\right)^{\hat{U}_0^2} + \left(\frac{T}{E_F}\right)^{1-2\hat{U}_0}\right]. \qquad (11a)$$

Equation (11a) shows that the conductivity increases as the temperature is lowered for $\hat{U}_0 < \frac{1}{2}$. In (11a) the last term represents the contribution from the $2K_F$ backscattering channel. In one dimension we obtain $(T/E_F)^{-2\hat{U}_0}$, showing clearly that when T decreases the conductivity decreases. In two dimensions the $2K_F$ backscattering term is reduced by a factor of $1/N$. As a result, we obtain

$$\frac{1}{N}\left(\frac{T}{E_F}\right)^{-2\hat{U}_0} \simeq \left(\frac{T}{E_F}\right)^{1-2\hat{U}_0}.$$

The extra factor $1/N$ is due to the fact that for each $2K_F$ backscattering channel we have N non-backscattering channels.

At low temperature, (11a) is not valid since localization will take place. If n_i is the impurity concentration, they remain uncorrelated at short distances, $d \leqslant d_i = 1/\sqrt{n_i}$. As a result (11$a$) will not be valid for $\ell = \mathrm{Log}(E_F/T) > \mathrm{Log}(d_i K_F)$. The uncorrelated impurity approximation will not hold at temperatures

$$T \leqslant E_F\left(\frac{\sqrt{n_i}}{K_F}\right) = V_F\sqrt{n_i} \equiv T_{loc}.$$

The results given in (11a) and the estimate of the onset of localization T_{loc} might be relevant to the recent experiments [5] since $T \geqslant T_{loc}$.

To conclude, we have considered the single impurity problem in two dimensions with interaction in the absence of spin and Cooperon fluctuations. We have shown that for finite temperatures we find a range of temperatures for which the interactions control the conductivity. For this case we observe that the conductivity increases with decreasing temperature.

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References

[1] Kane C L and Fisher M P A 1992 *Phys. Rev. Lett.* **68** 1220
[2] Oreg Y and Finkelstein A M 1996 *Phys. Rev. Lett.* **76** 4230
[3] Giamarchi T and Schulz H J 1988 *Phys. Rev.* B **37** 325
[4] Finkelstein A M 1983 *Zh. Eksp. Teor. Fiz.* **84** 168 (Engl. Transl. 1983 *JETP* **97** 83)
Finkelstein A M 1984 *JETP Lett.* **40** 63
[5] Kravchenko S V, Simonian D, Sarachik M P, Mason W and Furneoux J E 1996 *Phys. Rev. Lett.* **77** 4938
[6] Luther A 1979 *Phys. Rev.* B **19** 320
[7] Haldane F D M 1992 *Helv. Phys. Acta* **65** 152
[8] Schmeltzer D 1993 *Phys. Rev.* B **47** 11 980
Schmeltzer D 1994 *Phys. Rev.* B **50** 12 733
Schmeltzer D 1995 *Phys. Rev.* B **51** 370
Schmeltzer D 1996 *Phys. Rev.* B **54** 10 269
[9] Schmeltzer D 1995 *Phys. Rev.* B **52** 7539
[10] Shankar R 1994 *Rev. Mod. Phys.* **66** 129
[11] Kopietz P 1997 *Bosonization of Interacting Fermions in Arbitrary Dimensions* (Berlin: Springer)
[12] Nitzan A, Freed K F and Cohen M H 1977 *Phys. Rev.* B **15** 4476